

Modulus

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$$

e.g. $|3| = 3$ $|-3| = 3$
 $-|3| = -3$ $-|-3| = -3$

$$f(x) = x^2 - x - 6$$

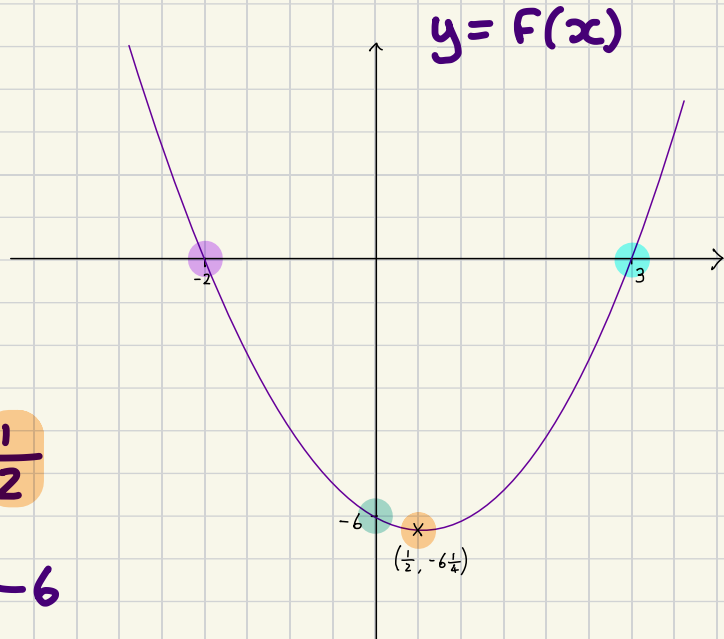
- $f(0) = -6$

- $f(x) = x^2 - x - 6$
 $= (x+2)(x-3)$

$$\Rightarrow f(x) = 0 \text{ at } x = -2, 3$$

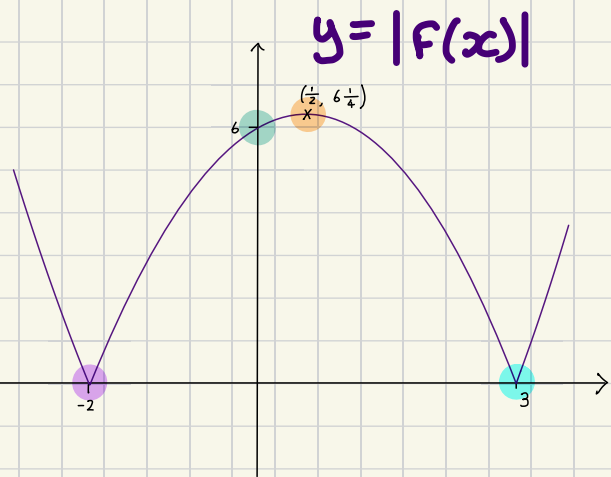
- Minimum at $x = \frac{-2+3}{2} = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{2} - 6$$
$$= -6\frac{1}{4}$$



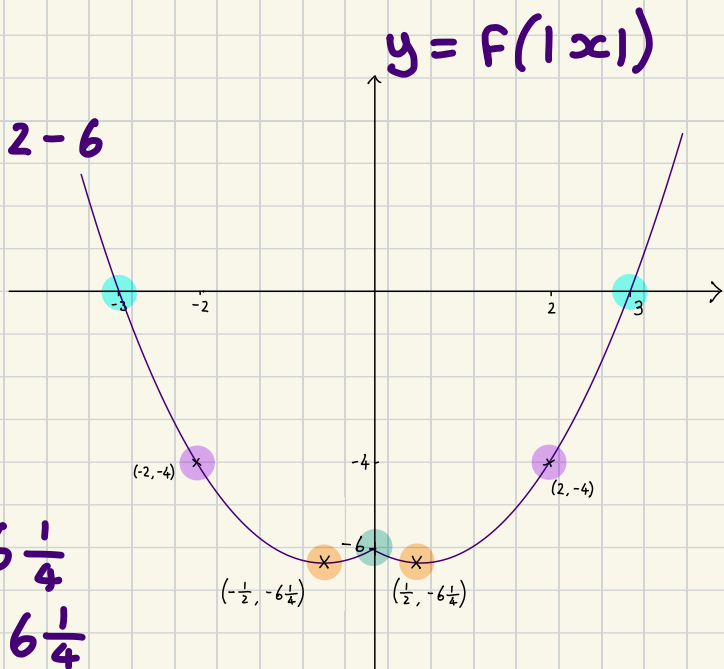
$$y = |F(x)|$$

- $x = -2 \Rightarrow y = |F(-2)| = |0| = 0$
- $x = 3 \Rightarrow y = |F(3)| = |0| = 0$
- $x = 0 \Rightarrow y = |F(0)| = |-6| = 6$
- $x = \frac{1}{2} \Rightarrow y = |F(\frac{1}{2})| = |-6\frac{1}{4}| = 6\frac{1}{4}$



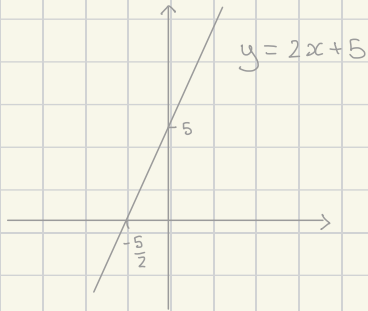
$$y = F(|x|)$$

- $x = -2 \quad y = F(|-2|) = F(2) = 2^2 - 2 - 6 = -4$
- $x = 2 \quad y = F(2) = -4$
- $x = 3 \quad y = F(|3|) = F(3) = 0$
- $x = -3 \quad y = F(|-3|) = F(3) = 0$
- $x = 0 \quad y = F(|0|) = F(0) = -6$
- $x = \frac{1}{2} \quad y = F(|\frac{1}{2}|) = F(\frac{1}{2}) = -6\frac{1}{4}$
- $x = -\frac{1}{2} \quad y = F(|-\frac{1}{2}|) = F(\frac{1}{2}) = -6\frac{1}{4}$

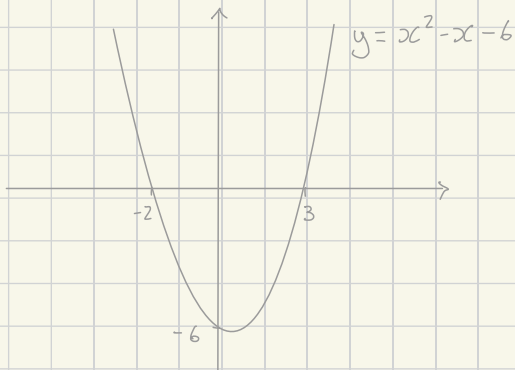
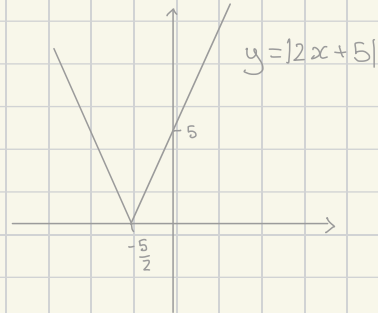


- Q a) sketch $f_1(x) = |2x+5|$ and $f_2(x) = |x|^2 - |x| - 6$
b) Find all intersections of these functions

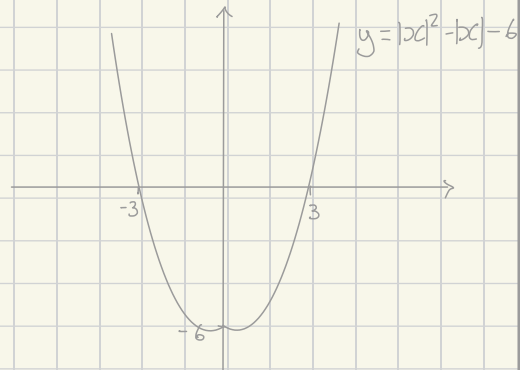
Rough Work



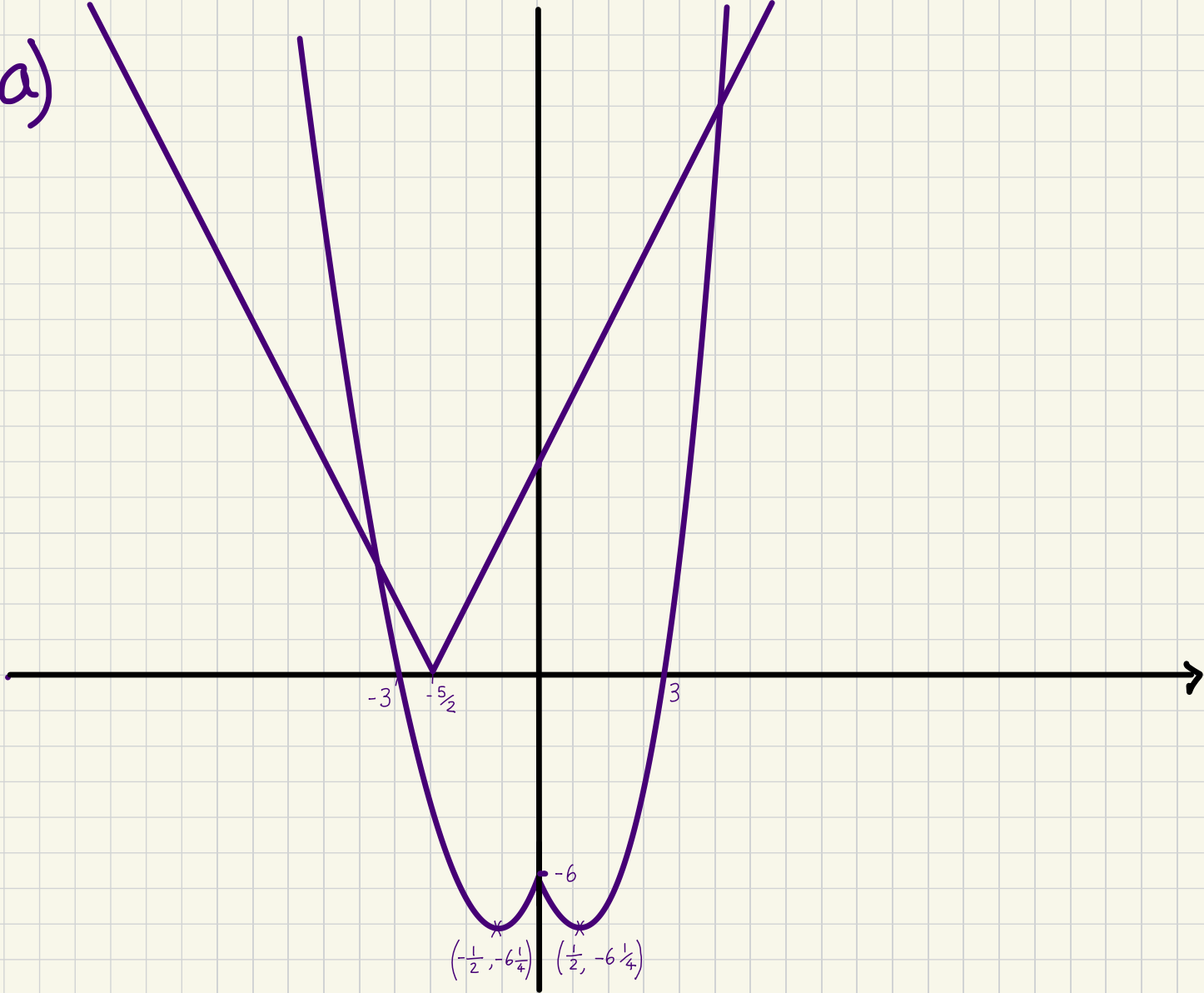
$\Rightarrow$



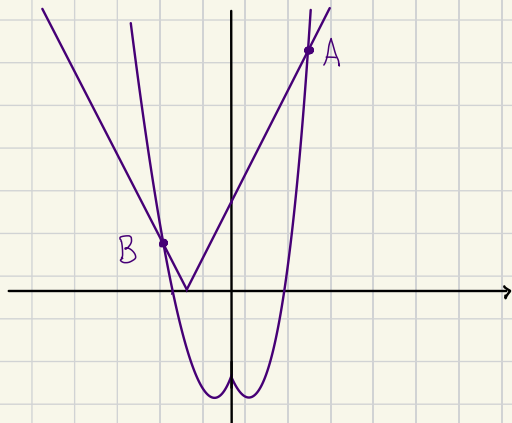
$\Rightarrow$



a)



b)



As we can see from the graph there are two pts of intersection which we label A and B

At A x is +ve

$\Rightarrow f_1(x) = |2x+5|$ is the same as $y = 2x+5$

$f_2(x) = |x|^2 - |x| - 6$ the same as $y = x^2 - x - 6$

$\Rightarrow$ At A $2x+5 = x^2 - x - 6$

$$\Leftrightarrow x^2 - 3x - 11 = 0$$

$$\Leftrightarrow x = 5.14 \text{ or } -2.14 \quad (3 \text{ s.f.})$$

So at A $x = 5.14$ $y = 2(5.14) + 5$

$$\boxed{A = (5.14, 15.28)}$$

At B x is -ve

$\Rightarrow f_2(x) = |x|^2 - |x| - 6$ is the same as

$$y = (-x)^2 - (-x) - 6$$
$$= x^2 + x - 6$$

At B $x < -\frac{5}{2}$ and so

$\Rightarrow$ at B $f_1(x) = (2x+5)$ is the same
as $y = -(2x+5)$

(because $|2x+5| = \begin{cases} 2x+5 & \text{if } 2x+5 \geq 0 \\ -(2x+5) & \text{if } 2x+5 \leq 0 \end{cases} \Rightarrow x \leq -\frac{5}{2}$)

at B $x^2 + x - 6 = -(2x + 5)$

$$\Rightarrow x^2 + 3x - 1 = 0$$

$$\Rightarrow x = 0.303 \text{ or } -3.30 \text{ (3 s.f.)}$$

$$\vec{B} = (-3 \cdot 30, 1 \cdot 61)$$