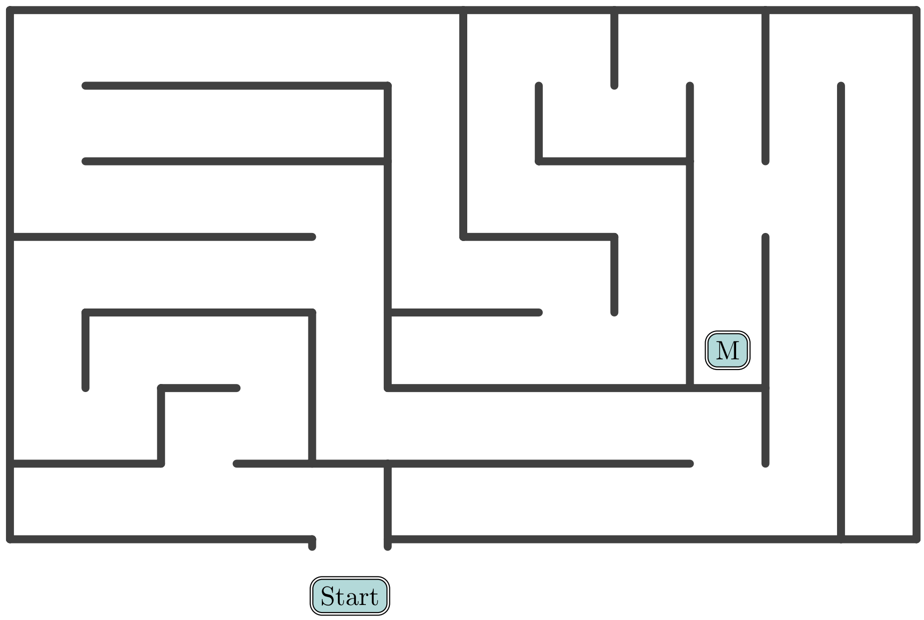
Mazes, Minotaurs and Maths

In Greek mythology The Minotaur was a half human half bull beast that had been trapped within a maze below Crete. After several years of terrorising the people who lived nearby The Minotaur was finally defeated by Prince Theseus. The main challenge for Theseus was finding his way through the maze without getting lost.

The paths within a maze can be untangled using networks, an area of maths that uses diagrams rather than numbers, and feels more like solving a puzzle than a maths problem. We'll show how we can construct networks from mazes, discuss some real world applications and then try to solve some examples ourselves either independently or in small groups.

Here's an example of a maze.

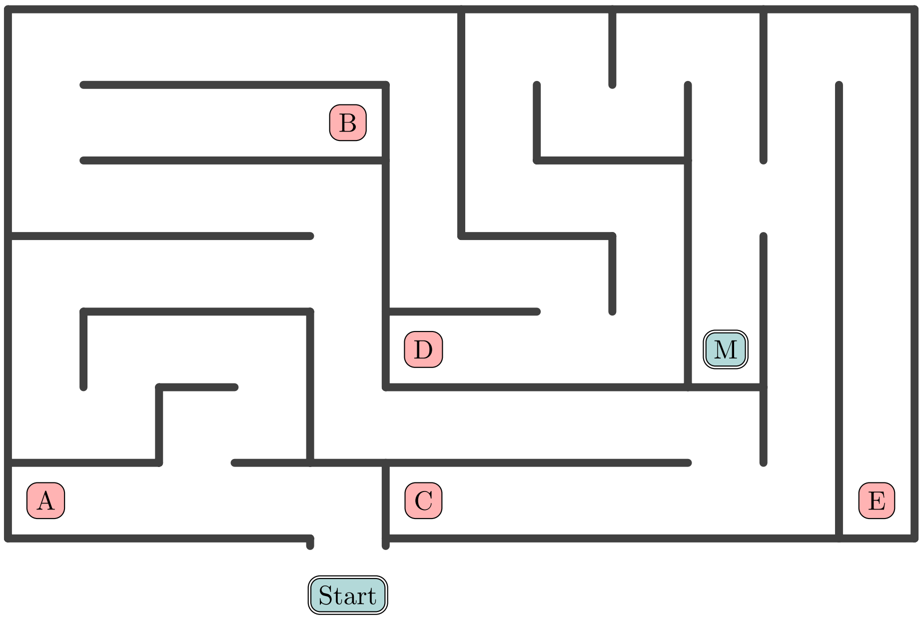


I'm sure you can all solve it pretty easily! But what if I asked you how many solutions there are (without having to backtrack)? Could you give a clear explanation of why those were the only possible solutions? Maybe there's a nicer way to represent the information given in the maze.

We're going to form a graph. Not the usual graph you think about but this is going to be a *graph* with *nodes* which we'll label by letters, and *edges* which are lines joining the nodes.

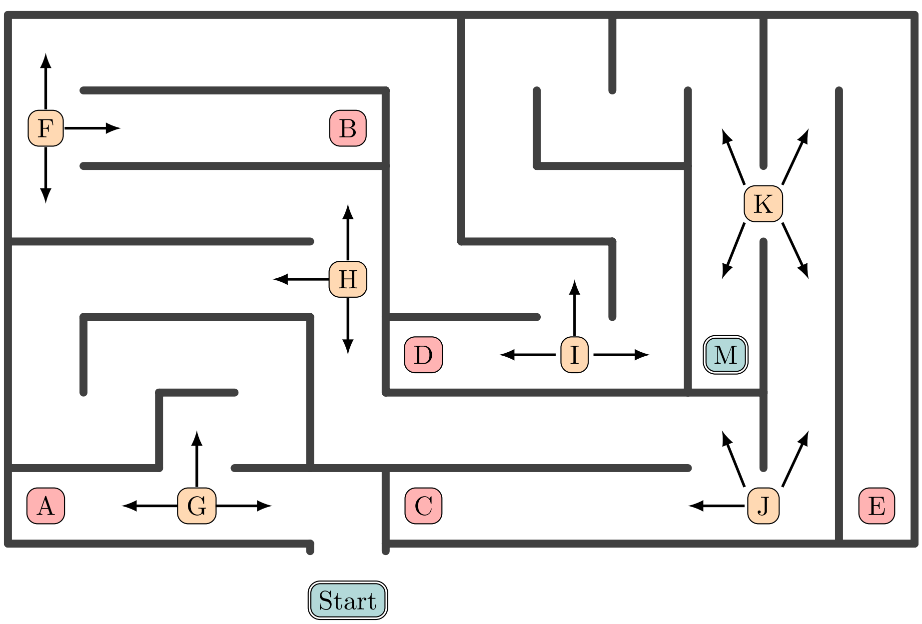
Let's label all the dead ends with red letters.

So we all use the same labels lets start at the bottom left, work up and then along.



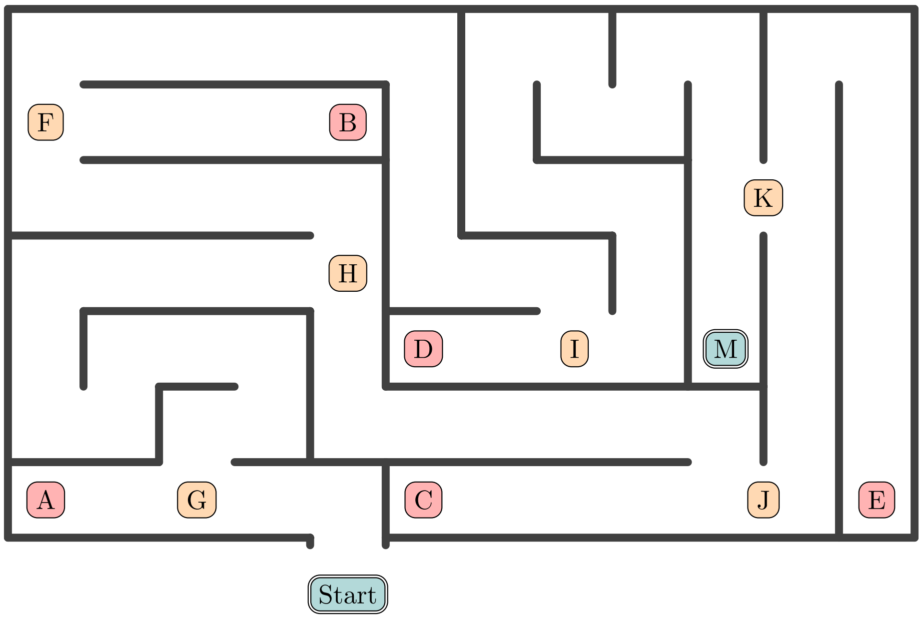
Let's label all the places where we have a choice of direction to make with orange letters.

Using the same labelling convention as with the dead ends.



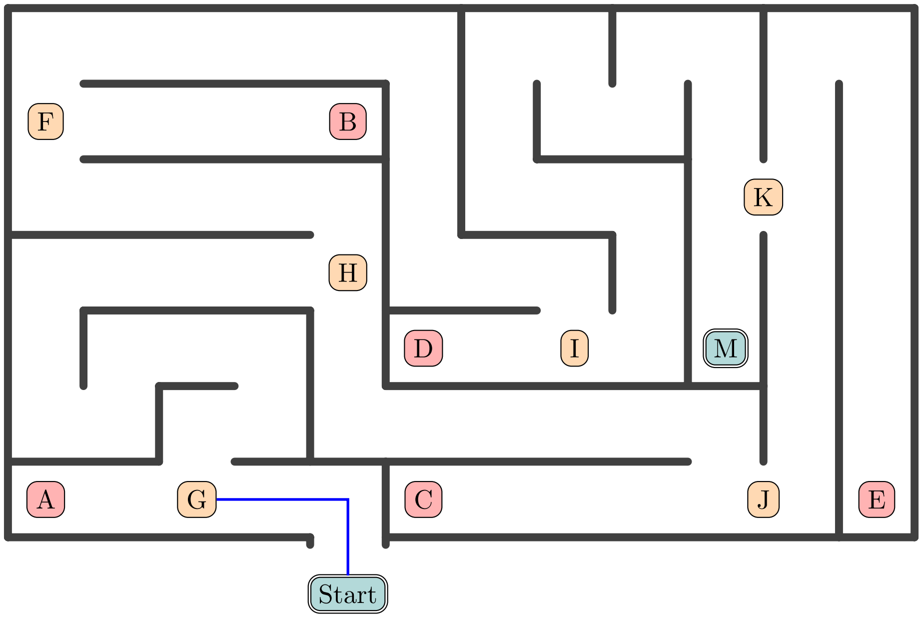
Here are all our labels!

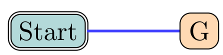
We're going to use them to rephrase the maze.



Veronica Kelsey Source info: plus.maths.org/content/maths-amazes

Walking from the start the first letter we reach is G.





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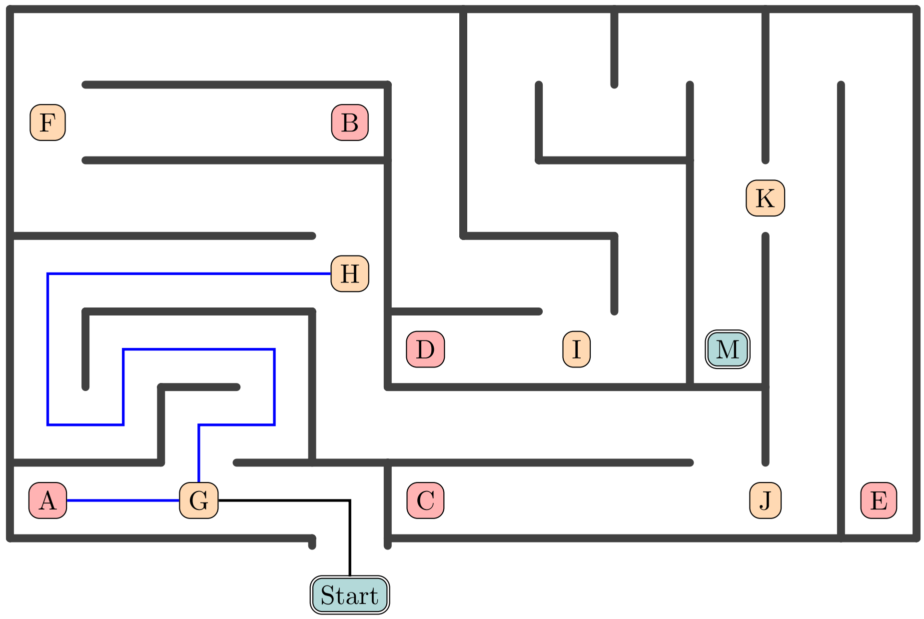
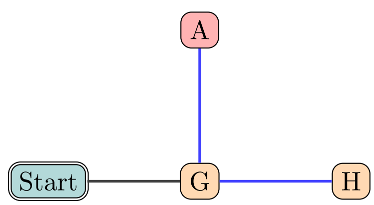
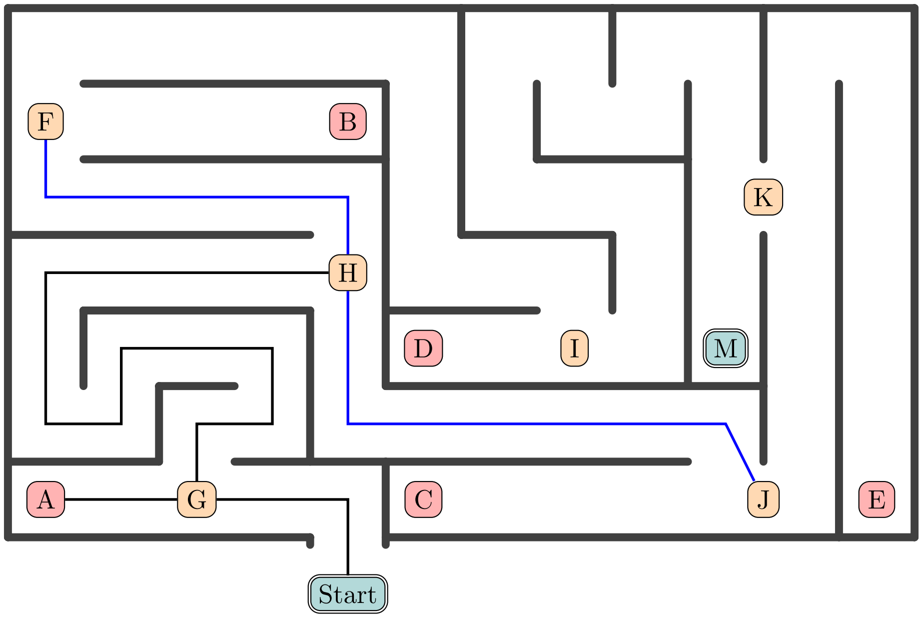
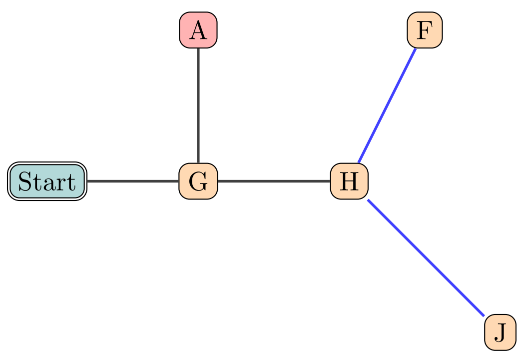
Then the two paths from G lead to A and H.



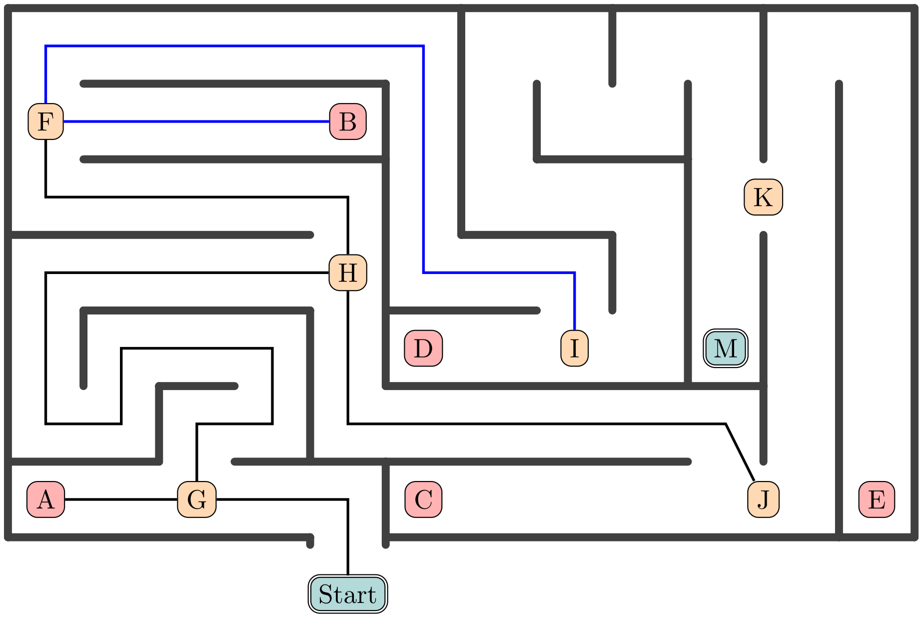


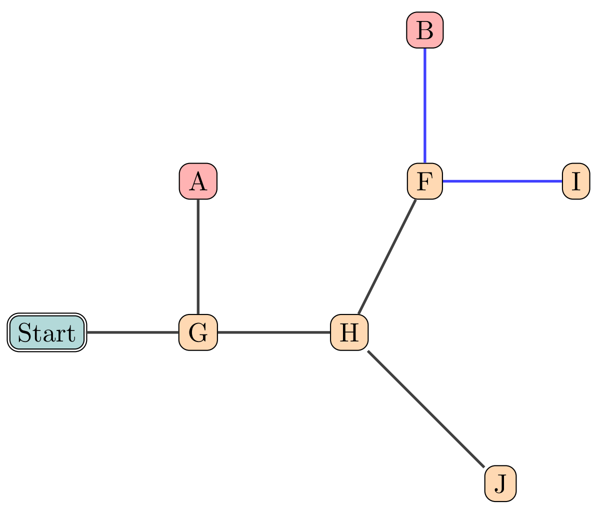
The two paths from H lead to F and J.



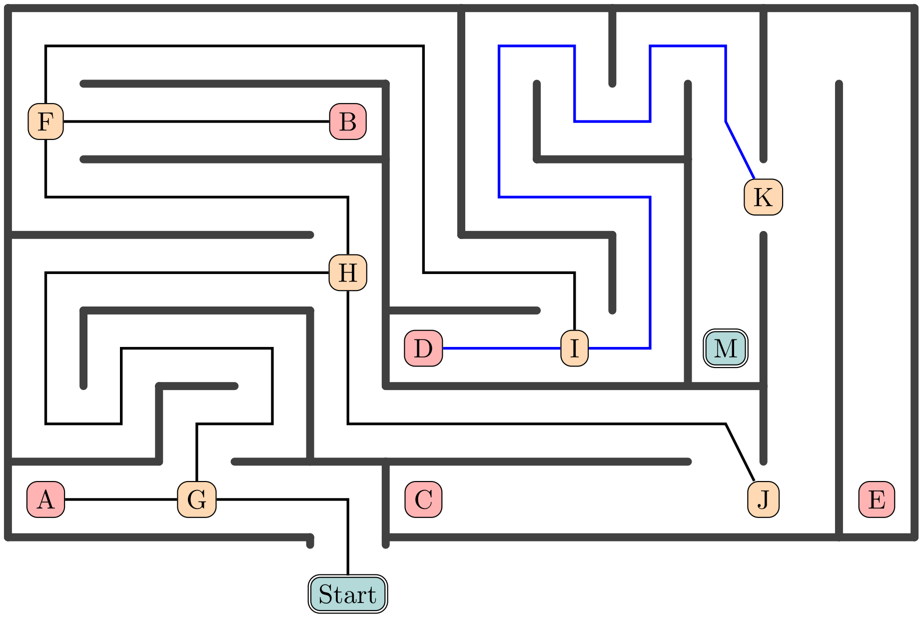


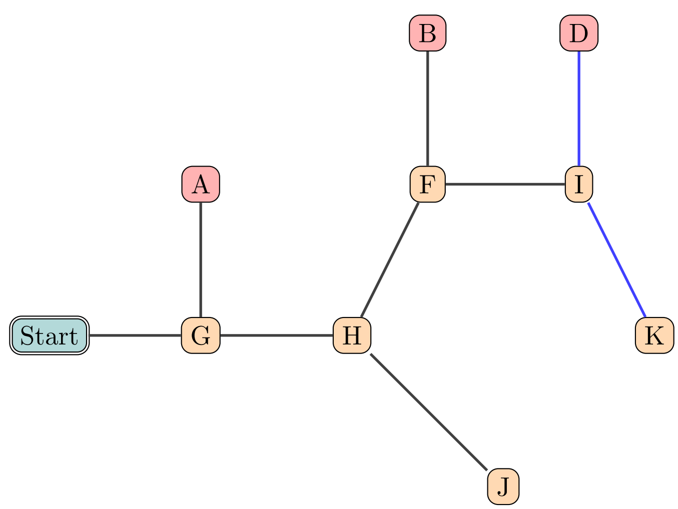
The two paths from F lead to B and I.



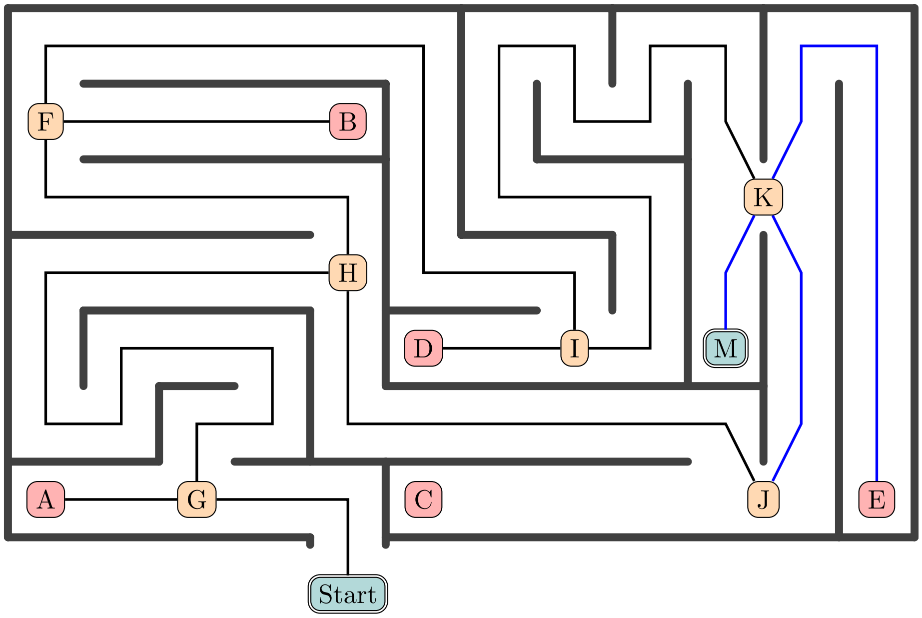


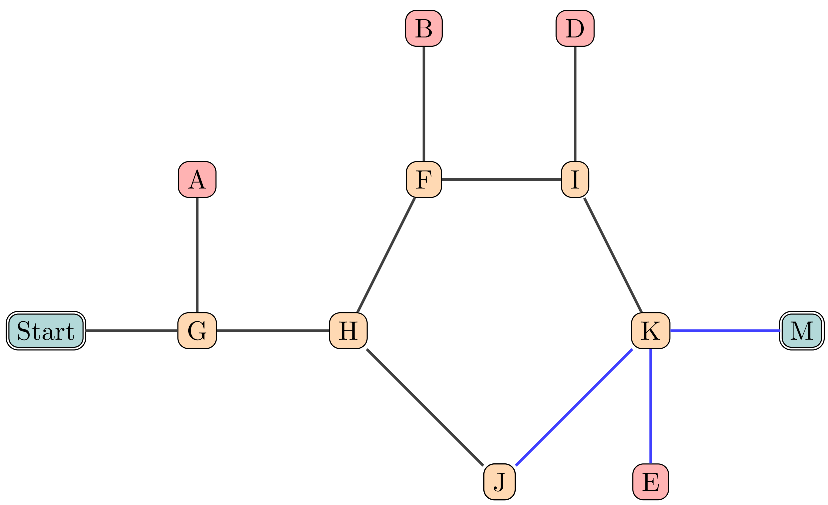
The two paths from I lead to D and K.



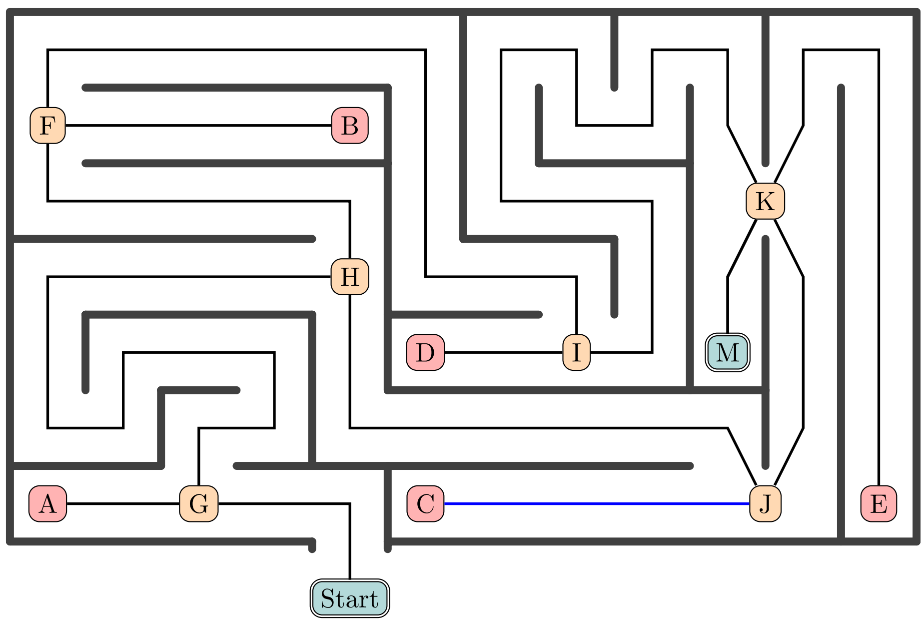


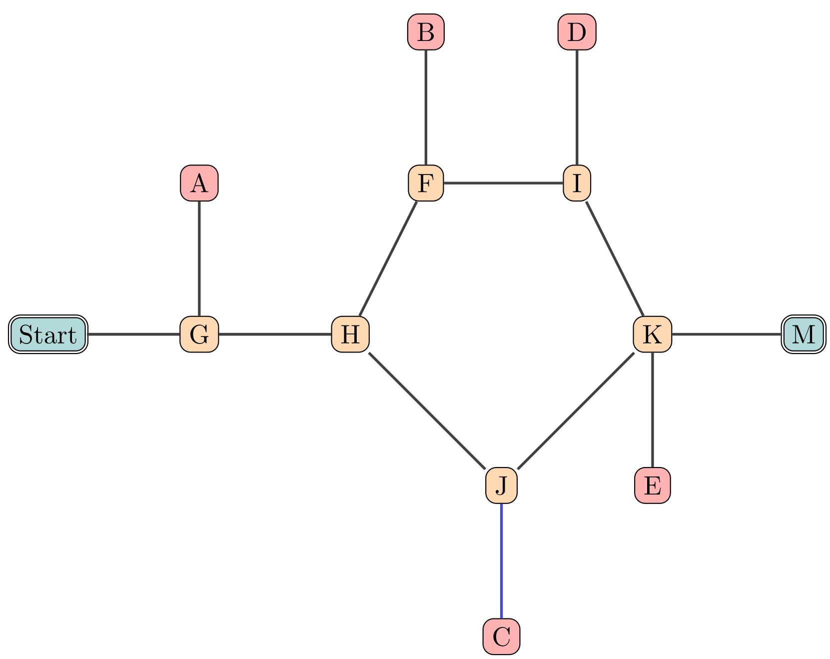
The three paths from K lead to M, J and E.





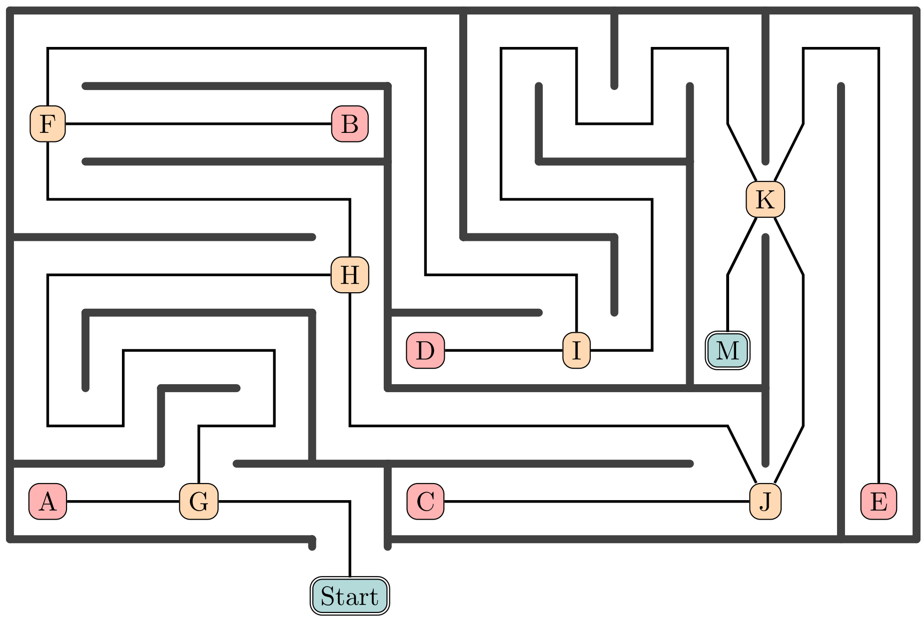
The two paths from J lead to C and H. Since C and H are already on the diagram we just join J to them rather than writing them down again.

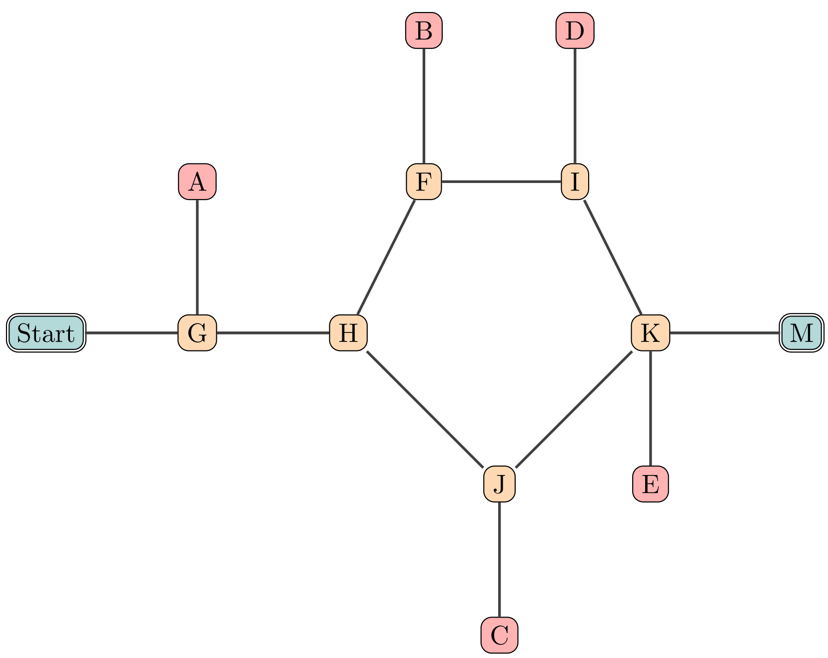




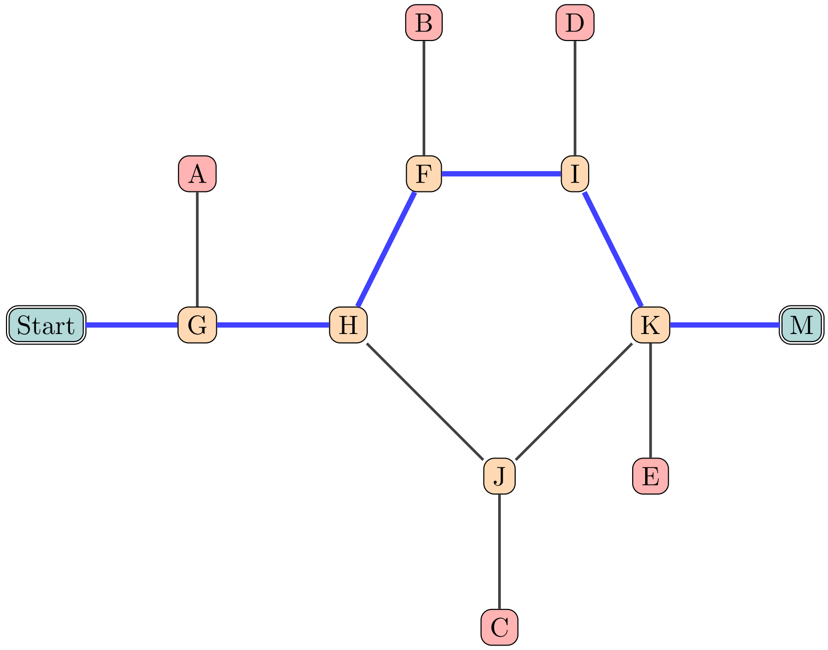
We can see we've travelled along all paths in the maze and so we've gathered all the information in the diagram below.

The red letters only have one line coming out of them, this corresponds to the fact that our only option when we hit a red letter is to retrace our steps. The number of lines coming out of the orange letters corresponds to the number of choices of direction there were at that junction.

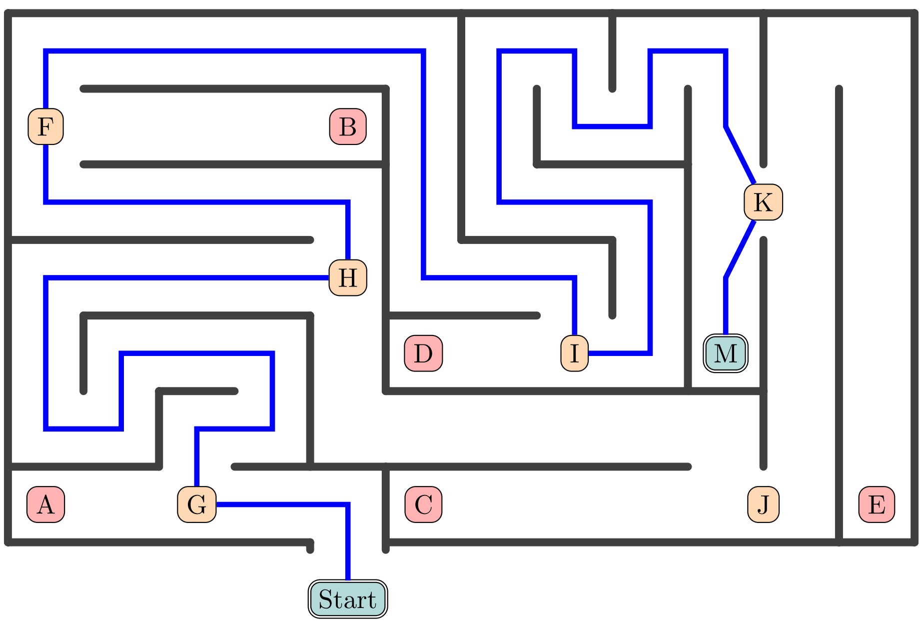




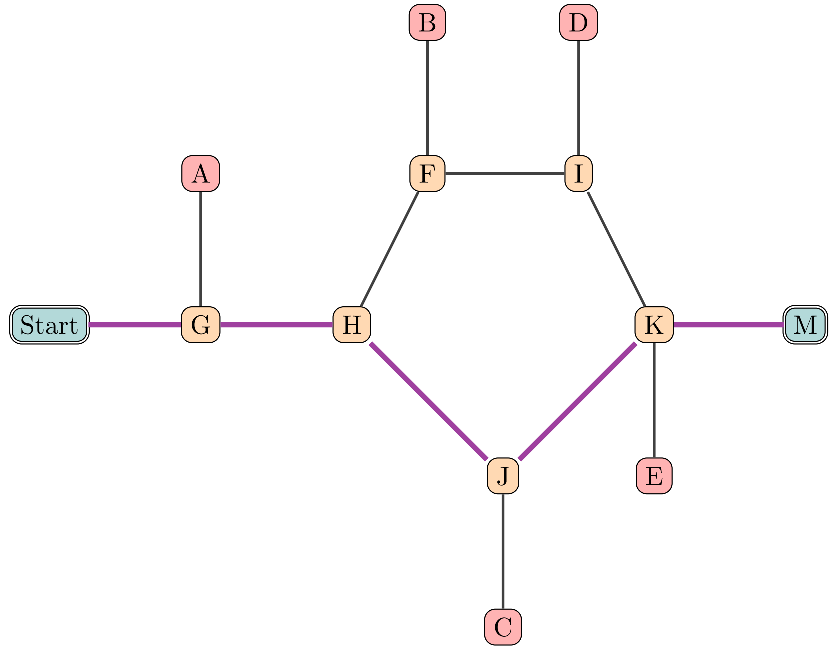
It's easy to see that there are two routes in the diagram. Here's one shown in blue.



This is what it looks like in the maze.



Here's the other route in purple.



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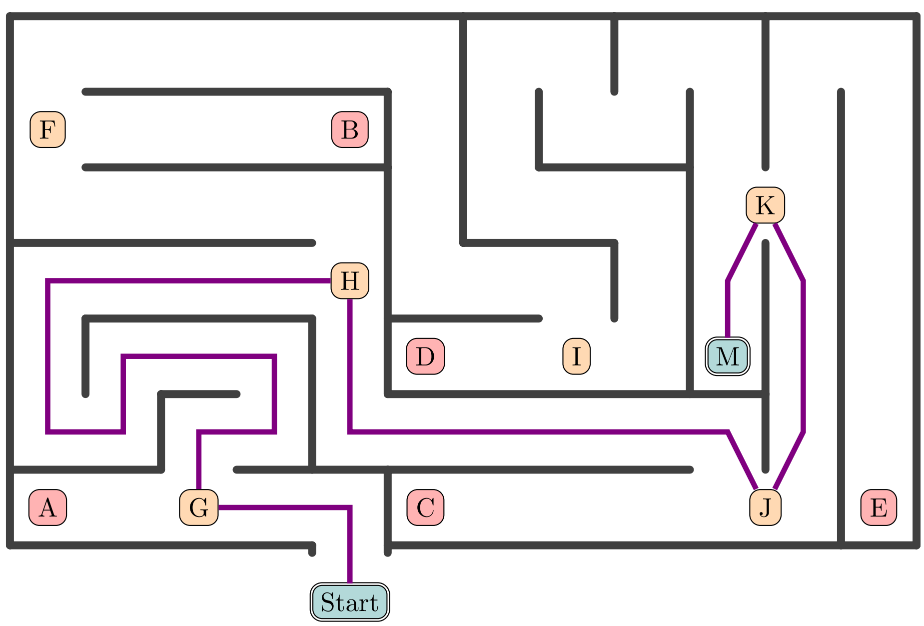

As I said at the start, this maze wasn't too tricky. But if we had a huge mazes or a 3D maze then working with the network is much easier. It's also much easier to store a network on a computer because it removes all of the "noise" of a maze and leaves only the important info.

Where is this used?

1. Tube networks - we use them to find the right route from one place to another. TFL use them to identify weak nodes in the network - points where a failure at one station would have large knock on effects for the whole network. In our worked example weak nodes would be G,H and K, if there were removed then there would be no solution. If F was removed then we could take the lower path.

2. World wide web - encodes the information of how webpages link to one another. Used by search engines to rank the search results so the most useful one is at the top.