Group Recognition

- Given a set of elements $g_1, \dots, g_r \in S_n$ how do we find out what $G = \langle g_1, \dots, g_r \rangle$ is?

- In general this is hard
  * knowing $|G|$ isn't enough
    For example, we know there are 2 groups order 4 C_4 and $C_2 \times C_2$
    But it gets worse... there are 9310 groups of order 2187
  * calculating all elements of G is computationally expensive
  * knowing the possible orders of the elements of G isn't always enough to determine G
  * Theres lots of interesting research and results in this area

- Thankfully when $|G|$ is small things aren't so bad
- Heres a table for $|G| \leq 7$

$\lvert G\rvert$	$G \cong$	Element Orders	Abelian?
1	Trivial group	1^1	✓
2	C_2	$1^1, 2^1$	✓
3	C_3	$1^1, 3^2$	✓
4	C_4	$1^1, 2^1, 4^2$	✓
	$C_2 \times C_2$	$1^1, 2^3$	✓
5	C_5	$1^1, 5^4$	✓
6	C_6	$1^1, 2^1, 3^2, 6^2$	✓
	$S_3 \cong D_6$	$1^1, 2^3, 3^2$	X
7	C_7	$1^1, 7^6$	✓

2 group (up to iso) of order 4

The notation $a_1^{b_1}, a_2^{b_2}, \dots$ means G has b_1 elements of order a_1, b_2 elements of order $a_2, \dots$

p, prime	C_p	$1^1, p^{p-1}$	✓

up to iso only 1 grp order p

- How to spot a dihedral group?

$$D_{2n} = \langle r, s \mid r^n = s^2 = 1, \ srs = r^{-1} \rangle \qquad |D_{2n}| = 2n$$

Some people call this D_n since its the symmetry group of an n-gon